A Sphere As A Gnomon, or The Olives in a Toothpick, the Frog of Savian and the Matelica Sphere of Nicelli

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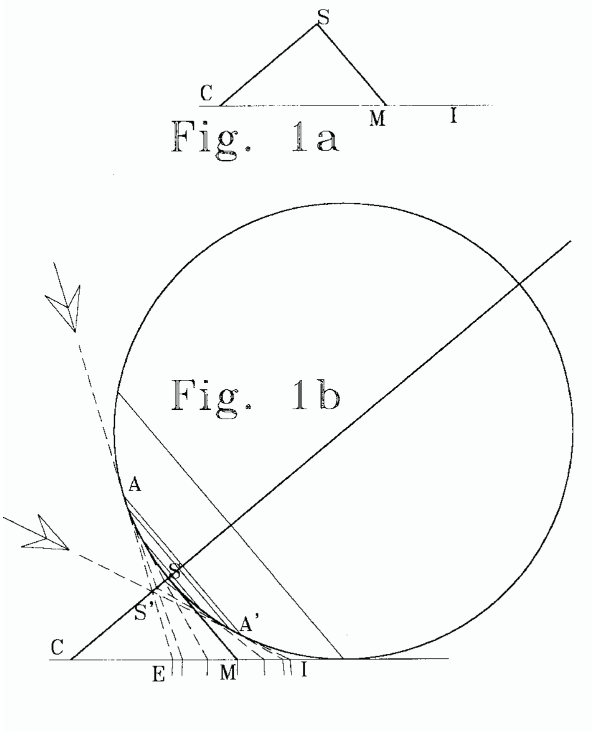
Perhaps these ideas have the most unusual origins. In an old film (I think "Sabrina") there was an elderly gentleman, who tried uselessly to remove the olive from a glass, doing unbelievable acrobatics. If only he had had a toothpick.... Or did he have one?... I don't remember.

I don't know why, but the idea has come to me from there. If the pick is a gnomon, and if the olive has more notable dimensions, e.g. those of a basketball, what happens when this is used as a sundial? Does something in the lines change?

The olives are like the cherries, one takes another, and I then started to widen the horizons, finding that the "lucubration" brings up the frogs of Savian, and the Nicelli essay on the Matelica sphere, in a Seminar three years ago. But I don't repeat here the matter of this talk; I only point out that the affinity exists.

I confined the problem to the case of a horizontal dial, with the polar gnomon and the sphere tangent to the plane of the dial.

As usual, I want to show that with exclusively graphic methods (working a little bit with pencil and imagination) we can accomplish the same job they (Fabio Savian and Alberto Nicelli) did with analytical methods. I hold to specify however that the analytical methods, once focused, offer outcomes that are much more rapid and precise than the methods I propose; but the graphic search allows us to focus on many "coincidences" that escape anyone who applies the ready-made algorithms.



Figures 1 and 2

Suppose CM is the horizontal plane, and CSM is the gnomonic triangle, with S the vertex of the gnomon (Fig. 1a). The resulting horizontal dial is illustrated by Fig. 2a. The image is extended to only one half of the plane, but it conveys the idea.

If, on the CS axis, we insert a sphere with its axis in S, some modest changes to the sundial face happen, but only for the declination lines.

We can see what happens in the figures 1b and 2b. In Fig. 1b we see that nothing changes in the equinox days, because the solar ray is tangent to the sphere in the S point; therefore the equinoctial line MP in Fig. 2b is identical to that of Fig. 2a.

Also the hour lines, depending exclusively on the polar stylus, are not modified.

The substantial modifications happen for the declination curves, because, *e.g.* to the solstices, the shadow of the sphere (really only the AA' ring, limit between the zone shaded and that illuminated, is

Fig. 2a

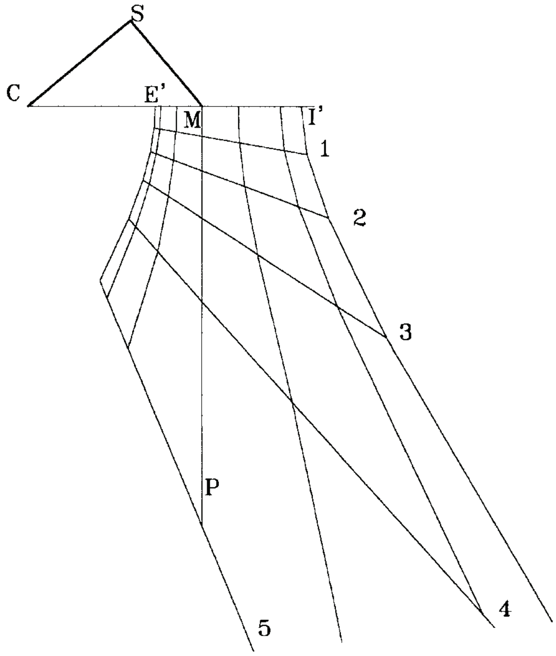
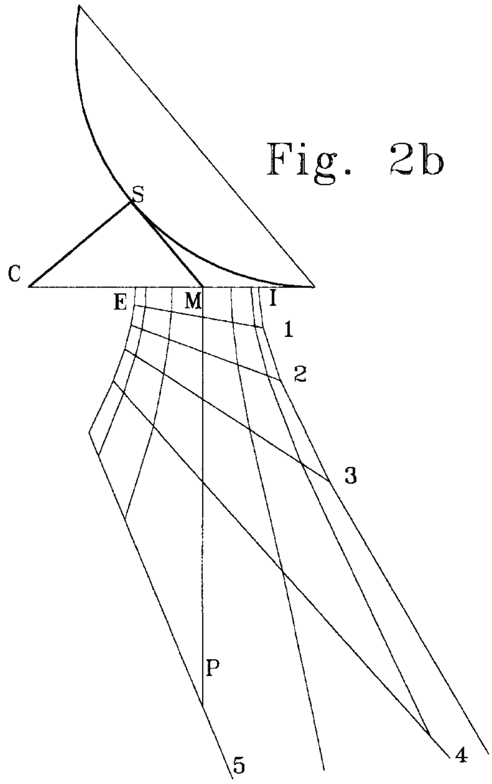


Fig. 2b

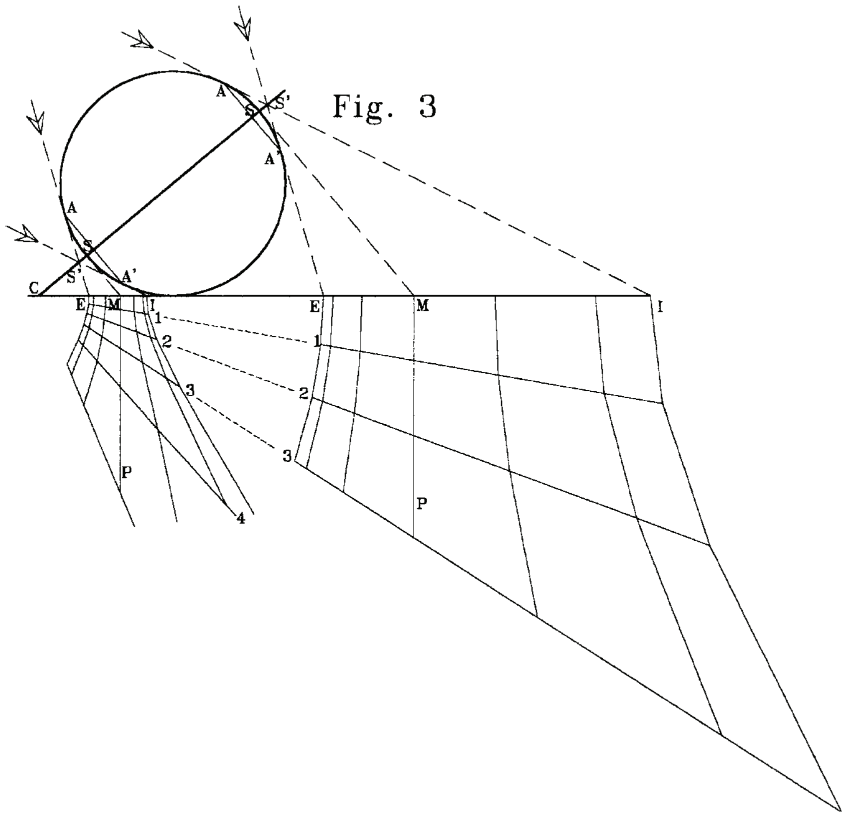


playing the fundamental role) "shortens" the gnomon, making the S' point become the vertex; so at noon E and I points, shadows of S', are nearer to C than E' and I' points of the Fig. 2a.

Therefore in Fig. 2b the declination lines (except the equinoctial line MP) are transferred toward C, so the intervals between them have widened in the summer and narrowed in the winter part. (beware: the moving is not proportional; unfortunately it is necessary to trace each one).

Obviously, it is possible to build the dial with only a small part of the sphere, like that in the Fig. 2b.

Fig. 3



<u>Figure 3</u>

If we perform the analogous construction for the other pole of the sphere (I have used the same letters to underline the analogy), we get a second clock, in which the hour lines are the prolongation of the Fig. 2 lines.

The declination lines suffer a transfer as regards the equinoctial line, but now toward the outside, to the contrary of that in Fig. 2b: E and I points are more distant, from the centre of the clock, than in a dial done with S gnomonic point, without the sphere. The "winter" part of the clock is widened, while the “summer” one has stayed narrowed.

Here the unhealthy idea goes off: if I remove the stick (pardon, the polar gnomon), how could I design the clock? Where do I trace the hour lines, what are the declination curves?

Savian and Nicelli, help…

The matter presents any initial difficulty, because one begins to think that the shadow of the sphere is more "wide" in the extreme hours than at midday, and is longer in winter than in summer... In short the problem gives the impression of being rather complex, not easily handled. Many fellows surrender, or they ask for the algorithm of the fellows above.

Really the problems are simpler than they appear to first sight: also if we take away the polar gnomon, the shadows of the sphere continue to depend on the AA' tangency circles, and therefore the declination curves can be traced as if the gnomonic points, underlined in Fig 1b, materially exist. Therefore Fig. 4 is the same as Fig. 3 but with the hour lines removed.

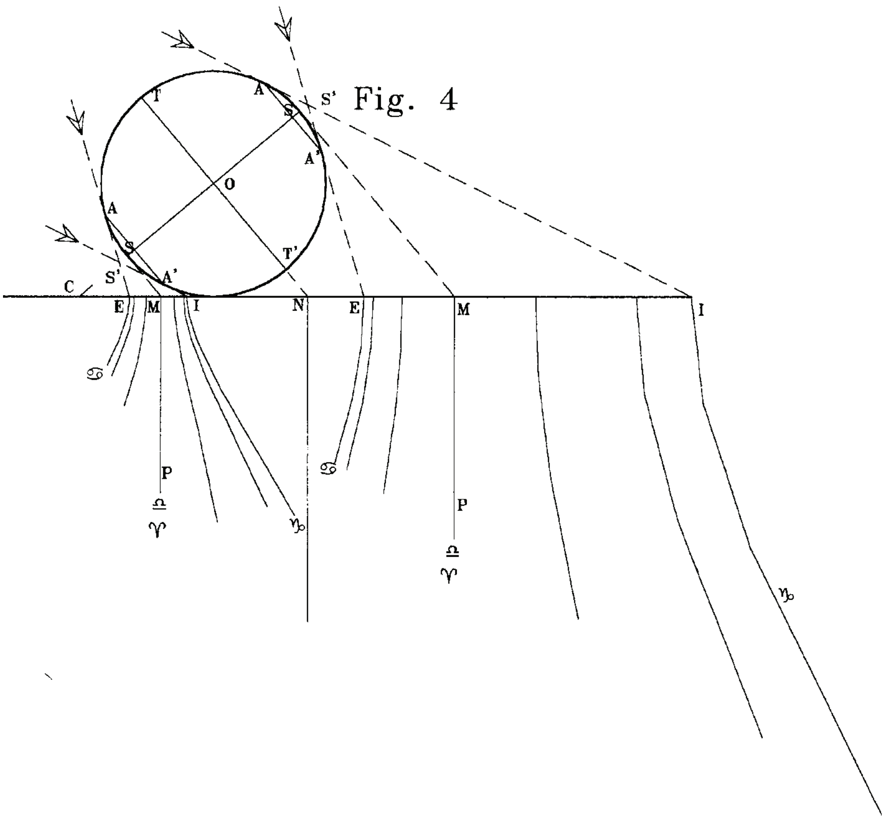


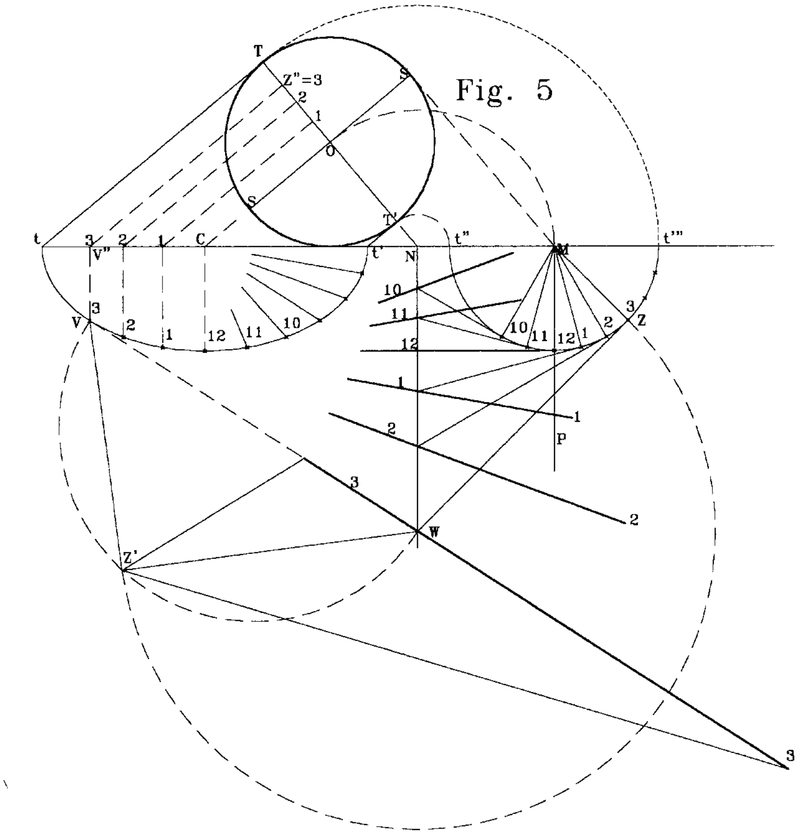
Fig. 4

To better spread: on the summer solstice day, the shade of the sphere will always be to the inside of the two curves coming from the two E points, always tangent to them; on the winter solstice day, they will be tangent to the hyperbolas coming from the I points, and so on for the other declination curves.

Let us see what happens to the hour lines: Fig. 5.

Imagine that the sphere is rolled by a parallel cylinder to the polar axis: it is tangent to the sphere along the TT' circumference and crosses the dial plane as a "slice of salami"; the extreme points of intersection are t, t’ points, and C/12 width is the ray of the sphere. We are about an ellipse.

At noon the sun is above the cylinder, so the CO line (not the central one, but the two generatrixes to the sides of the cylinder) is the separation line between the side surface in light and that in shade. I could tell that now the CO line is the gnomon, and O is the gnomonic point. Therefore the shadow of O moves ahead and back on the same hour plane according to the seasons, and therefore determines the length of the noon line.



One hour after, at one o'clock, the generatrix 1/1 of the cylinder will become the gnomon. And also here the hour plane will always be the same, regardless of the season. And similarly for the other hour lines.

Fig. 5 illustrates how to find the hour lines at one side of the sphere, finding the equinox points on the NW fictitious equinoctial line, and the cross points of the generatrixes of the cylinder on the ellipse. The theoretical scheme is only illustrated for the 3 hour line:

- capsize the TT' circle in t"t"' and on it find the hour points, remembering that the 12 point is on the MP line (*that, by chance, is the same line of Fig. 4.*)

- trace the cylinder beginning from TT', and find the tt' ellipse, whose central point is point C of Fig. 2.

- On the ellipse the hour points can be found making the turnovers, beginning from the capsized TT' circle (*with a middle school technique...*), or directly, as for the analemmatic dial. A third possibility also exists: if we transfer the Fig. 2 hour lines, they cross the ellipse in the desired hour points (*this is one of the many "coincidences"*).

- W point is found doing the perpendicular line to MZ, because it is the trace of the tangent hour plane to the cylinder.

- The 3 hour line is obtained by tracing the VW straight line.

- If now we want the extreme points of the hour line, we must get the VWZ' right-angled triangle, in which VZ'= V"Z" (length of the generatrix of the cylinder), and WZ'= WZ. The other two lines from Z' form the angle of maximum declination of the ecliptic as regards the equinoctial line Z'W.

This is a "normal" graphic construction, known to all the dialists, on which I do not dwell. The job must be repeated for each of the traced hour lines.

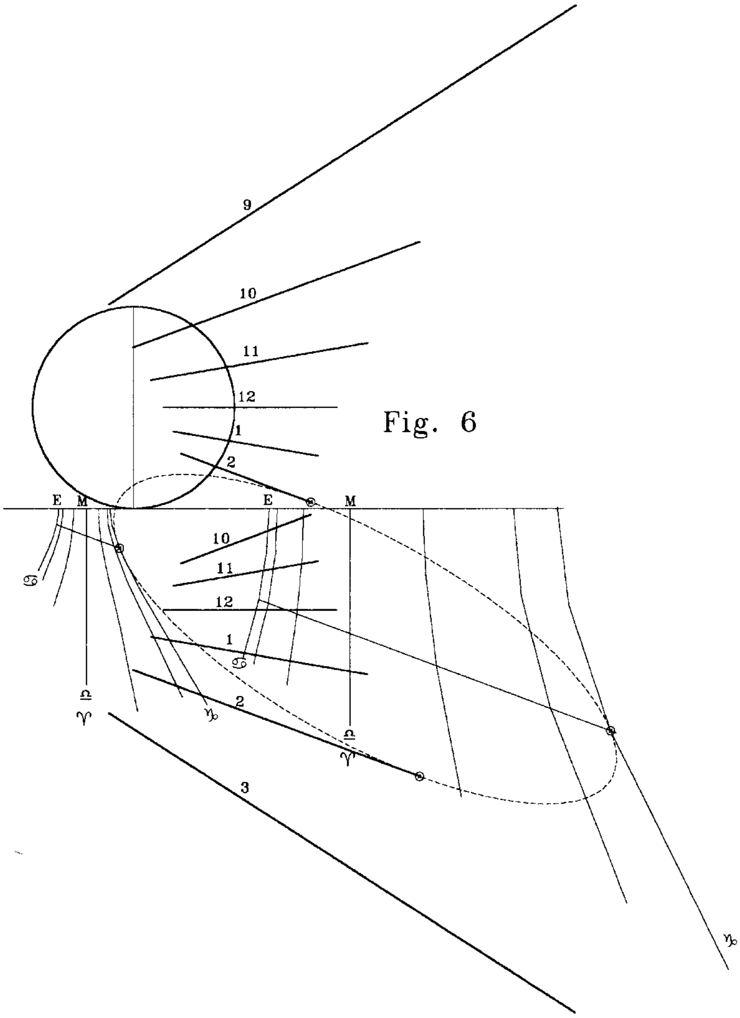


Fig. 6

Upshot

Figure 6 concludes this article, with the sketch of the final form of the clock: it is the overlap of the lines obtained in Figs. 4 and 5.

Observe that, obviously, the hour lines to the West of the sphere are the same as those to the East, with changed numbers.

The lines with the same number are parallel; but there is more to say: if we should insert on the graph the hour lines of Fig. 3, also these would be parallel to the lines with the same name. This consideration could be a reason to rethink the whole process, because we could find some simplification in the job of tracing Fig. 5.

In the figure, *e.g.*, the form of the winter solstice 2 hour shadow has been traced: the shade is tangent to both the 2 hour lines in their extreme points, and besides is tangent to the declination hyperbolas in their cross points with the 2 hour line obtainable from the shade of the axis of the sphere (that is the 2 hour line of Fig. 3).

Finally, one of the two focuses of the ellipse is the point in which the sphere is tangent to the quadrant (Dandelin's Theorem: If a cone is intersected by a plane in a conic, then the foci of the conic are the points where this plane is touched by the spheres inscribed in the cone); and the two segments joining the four opposite tangency points, mutually cut themselves in the middle point, thus finding the center of the ellipse.

I save the reader other considerations (*always the usual "coincidences"*) of a geometric character, arising out of the other mutual characteristics of the points of tangency of the shade ellipse. These "coincidences" would have some interest for the graphic construction of the shadows.

There is inside too much projective geometry; *Est modus in rebus.*

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